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THE HEDGING EFFECTIVENESS OF THE OPTIMAL HEDGE RATIO FOR CRUDE OIL, GASOLINE AND HEATING OIL

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Crude oil and refinery fraction prices are volatile. The literature has suggested hedging as one of the means to address such volatility. Most methods for hedging can be categorized as Ordinary Least Squares (OLS) based, Cointegration based, or Volatility based.
This study sought to compute and compare various methods for the optimal hedge ratio (OHR) for crude oil, gasoline and heating oil.Data on the spot and futures prices of crude oil, gasoline and heating oil were obtained from the Energy Information Administration (US EIA) database. The optimal hedge is estimated by the methods of OLS, Cointegration based Regression, Volatility based Regression, and the Kalman Filter to facilitate a comparison of the effectiveness of the hedging methods employed. This study makes a contribution to the literature as it proposes a new method to assess hedging effectiveness based on changing the OHR into a portfolio weight and minimizing the variance of the portfolio.

Keywords: Hedging Effectiveness, Optimal Hedge Ratio, Petroleum Prices

INTRODUCTION

From June 2014 to January 2015, the price of crude oil collapsed from US $105.79/bbl to US $47.22/bbl, a 55.36% decline. Such sudden decline in oil prices is not a new phenomenon with oil prices. In 2008, oil prices experienced a sudden decline in response to the global economic recession. It declined from its highest peak of US 136.49/bbl in June 2008 to US $37.43 in January 2009, from US $70.5/bbl in August 2006 to US $54.5 by January 2007, and from US $38/bbl in October 1990 to US $17.43 in February 1991. Even in the 1980s there were periods when oil prices experienced sudden and rapid decline (US EIA 2016). Such decline in oil prices can result in decline in revenue for oil producers.

In the literature, one method that has been identified to offset the risk of adverse commodity and asset price movement is hedging with futures. A futures contract is an agreement between two parties to buy and sell a given amount of a commodity at a specific price and location (Chang et al. 2011). Futures contracts should not be confused with forward contracts.A forward contract is also an agreement between parties to buy or sell a commodity or asset at a specific time at a given price. However, futures contracts are standardized and exchange-traded. Forward contracts are private agreements between two parties; they tend to be specialized/ customized, and carry a possibility of default. Futures contracts also carry no risk of default because the exchange acts as a counterparty, guaranteeing delivery and payment (Bacon and Kojima 2008; Chang et al. 2011). Futures contract are widely used because of their high liquidity, speed, and low transaction costs. Moreover, they are used frequently in the oil and refinery fraction industry since that industry has high price risk.An oil or refinery company would consider the hedging with futures if they want to reduce their oil price risk. If the government of a petroleum exporting country do not directly sell crude oil and refinery fractions, then the government would not engage in hedging. The hedging would only be performed by a company selling the crude oil or refinery fractions (Bacon and Kojima 2008).

Any economic agent considering hedging with futures must decide on the optimal amount of output to hedge due to the problem of basis risk.Basis risk is the risk that there can be excess loss or gains from hedging because of the imperfect negative correlation between the underlying and the hedge asset (Haushalter 2000; Knill et al. 2006). Two asset prices (spot A spot market is one in which the economic agent pays for the asset or commodity, and expects the commodity to be delivered immediately and futures) may not be 100% perfectly correlated. Therefore, a hedging strategy will not be 100% effective since the price changes will not be in entirely opposite directions from each other. In the case of spot and futures, their prices may not converge at the maturity of the futures. This imperfect correlation or basis risk creates the potential for a hedging strategy to cause excess gains or losses.

Several methods have been used to empirically compute the optimal hedge. Such methods generally fall in the following categories: (a) Ordinary Least Squares (OLS);(b)Cointegration based models; and(c) Volatility models.

The objective of this study is to compute and compare different methods that can be used to compute the optimal hedge ratio (OHR) for crude oil, gasoline and heating oil. This study makes a contribution to the literature as it proposes a new method to assess hedging effectiveness.

This study is structured as follows: Section two provided a literature review of hedging, section three reviewed the data that was used to compute the OHR. Section four reviewed the methodology used to estimate the OHR. Section five presented the results. Section six presented the conclusion.

LITERATURE REVIEW

Johnson (1960) explained the mechanics of hedging and sought to apply portfolio theory to reform the hedging theory. If an economic agent has created a portfolio of spot and futures, then the optimal hedge ratio is the ratio of the amount of spot and futures that will ensure that the value of a portfolio does not change (Hatemi-J and Roca 2006).

$$V_h = Q_s S - Q_f F \quad (1)$$
$$\Delta V_h = Q_s \Delta S - Q_f \Delta F \quad (2)$$

where V_h is the value of the portfolio that has been hedged, Q_s is the quantity of spot, Q_f is the quantity of futures, S is the spot price, F is the futures price, and Δ is the change in one of the variables.

Equation (1) was converted into changes in equation (2) because the source of uncertainty is the price. The change in spot prices from one period to the next is given by ΔS. The change in futures prices from one period to the next is given by ΔF. $\Delta S = S_t - S_{t-1}$and $\Delta F = F_t - F_{t-1}$.

The objective of the hedging strategy is that there should be no change in the value of the portfolio. This can be represented by settingV_h to 0. Then:

$$\frac{Q_f}{Q_s} = \frac{\Delta S}{\Delta F} \quad (3)$$

Let the optimal hedge, $h = \frac{Q_f}{Q_s}$, therefore $h = \frac{\Delta S}{\Delta F}$

Therefore, the hedge ratio h can be obtained as the gradient parameter in a regression where the spot price of a refinery fraction is regressed on the futures price of refinery fraction.

Substituting equation (3) into equation (2)

$$\Delta V_h = Q_s(\Delta S - h\Delta F) \quad (4)$$

The optimal hedge ratio will be the one that minimizes the risk of the possible change in the value of the portfolio. The risk can be measured by the variance of equation (4). This is given by:

$$variance(\Delta V_h) = Q_s^2(\sigma_s^2 + h^2\sigma_f^2 - 2h\rho\sigma_s\sigma_f) \quad (5)$$

whereσ_s^2 is the variance of ΔS, σ_f^2 is the variance of ΔF, ρ is the correlation between ΔF and ΔS. To minimize the variance of ΔV_h with regards to h, a partial derivate is performed:

$$\frac{\partial var(\Delta V_h)}{\partial h} = Q_s^2\left(2h\sigma_f^2 - 2\rho\sigma_s\sigma_f\right) = 0 \quad (6)$$

This results in an OHR defined by

$$h^* = \rho\frac{\sigma_s}{\sigma_f} \quad (7)$$

Ederington (1979) improved on Johnson (1960) formulae for hedging effectiveness. Hedging effectiveness is the percent reduction of the variance of the portfolio; it is defined by the formula

$$E = 1 - \frac{var(H)}{var\ (U)} \quad (8)$$

where$E$ is the hedging effectiveness, $var(H)$ is the variance of the hedge position (variance of futures returns), and $var(U)$ is the variance of the unhedged position (variance of spot returns).

Substituting the minimum variance X_f produces:

$$E = 1 - \frac{X_S^2\sigma_{\Delta S}^2(1-\rho^2)}{X_S^2\sigma_{\Delta S}^2} = \rho^2 \quad (9)$$

where ρ^2 is a square of the correlation coeffiicent of spot and futures price changes and $\sigma_{\Delta S}^2$ is the variance of spot price changes. In other words, R^2 from the regression of spot on futures prices is used as a measure of hedging effectiveness. Ederington (1979) MVHR assumes that investors are infinitely risk averse. While such an assumption is unrealistic, it was an improvement over the naïve hedging strategy. Thus, the first set of optimal hedge ratios which were estimated by Johnson (1960); Stein (1961); and Ederington (1979) was the coefficient from the OLS regression:

$p_t = \propto + \beta f_t + \varepsilon_t$ (10)

where p_t is the spot price, f_t is the futures price, $\propto$ is a constant, β is the optimal hedge ratio and ε_t is the error term of the regression. The beta (β) optimal hedge ratio performs similarly to the 1:1 naïve hedge strategy where the risk averse economic agent takes a futures position that is opposite to the spot position. Sometimes the OLS OHR method is referred as the Johnson-Stein-Ederington (JSE) method (Ji and Fan 2011).

Financial assets tend to be non-stationary. The estimation of parameters with non-stationary variables can lead to spurious regression results. However, if variables share a long term cointegrating relationship, parameters may be better estimated by the incorporation of an error correction term in the regression equation (Engle and Granger, 1987; Brooks, 2008). The estimation of the optimal hedge ratio via OLS methods that ignore the non-stationary and cointegrating relationship of spot and futures tend to result in the under estimation of the hedge ratio (Ghosh, 1993; Da-Hsiang 1996; El-Khatib, and Hatemi 2011). Ghosh (1993) also argued that OLS ignores lead-lag relationships contained within spot and futures prices, and may lead to misspecification. Myers and Thompson (1989) used a cointegration approach to estimate the optimal hedge. It is given by:

$\Delta p_t = \propto_p + \lambda \Delta f_t + \sum_{i=1}^{m} \beta_{pi} \Delta p_{t-i} + \sum_{j=1}^{n} \gamma_{pj} \Delta f_{t-j} + \theta_p z_{t-1} + \varepsilon_{pt}$ (11)

where Δp_t is the first difference of the spot price, $\Delta f_t$is the first difference of the futures price, z_{t-1} is the error correction term, λ is the estimate of the optimal hedge ratio that will minimize the variance θ.

Also, the presence of serial correlation in the residual, and the presence of heteroscedasticity lead to the inaccurate estimation of the optimal hedge ratio under OLS estimation (Park and Bera 1987; Herbst et al. 1993). An additional limitation of both OLS based and cointegration based methods for the derivation of the optimal hedge ratio is that such methods produce an OHR that is time invariant. However, it is widely accepted that financial assets returns, volatility, covariances, and correlations are time varying (Kroner and Sultan 1993). Other authors argued that if the joint distribution of spot and futures vary over time, then the estimation of constant hedge ratio would be inappropriate (Bollerslev 1990; Baillie and Myers 1991; Kroner and Sultan 1990).

Subsequently, some papers utilize Generalized Conditional Heteroscedasticity (GARCH) type models to capture the time varying properties while computing the optimal hedge. In such models, the OHR is specified as the ratio of the time varying covariance of spot and futures to the conditional variance of futures.

Chang et al. (2011) used the CCC-GARCH, VARMA-GARCH, DCC-GARCH, BEKK-GARCH and diagonal BEKK-GARCH to compute the OHR for Brent and WTI futures. They also computed the optimal portfolio weights, and used a hedging effectiveness index to compare the performance of the OHR estimated via the different models. They found that the diagonal BEKK was the best model for the computation of the OHR, while the BEKK was the worst model for computing the OHR. Hsu et al. (2008) used a GARCH with error correction model to compute the OHR; they found that this model improved the hedging effectiveness.

Wang et al. (2010) used a Kalman Filter error correction model (KF-ECM) to compute the OHR for nineteen stocks from the Taiwan stock exchange over the January 05, 1995 to February 28, 2009 period. In particular, they used the Kalman Filter to extract the common trend among Taiwan weighted stock index (TAIEX) and TAIEX futures. They then combined the common stochastic trend with an error correction model, and compared the OHR from the KF-ECM to the OHR derived from OLS, GARCH, and vector error correction models (VECM). They found that the OHR derived from the KF-ECM had a better hedging effectiveness than that derived from OLS, GARCH and VECM.

It must be notedthere is a dearth of information on using the Kalman Filter to compute the optimal hedge ratio for crude oil or its refinery fractions. This study intends to fill such gap.

To compare the hedging effectiveness of different volatility models, Ku et al. (2007) proposed the following hedging effectiveness index:

$HE = \frac{var_{unhedged} - var_{hedged}}{var_{unhedged}}$ (12)

where HE is the hedging effectiveness index, $var_{unhedged}$ is the variance of the spot or the unhedged position, and var_{hedged} is the variance of the hedged position. However, equation (12) is the same as equation (8).

Other authors have used equation (12) or (8) to assess hedging effectiveness (Andani et al. 2009; Zanotti at al. 2010; Yao and Wu 2012; Kumar 2014).

3.0 DATA USED FOR COMPUTING THE OHR

Data on WTI futures prices, gasoline futures prices and heating oil futures prices were obtained from the Energy Information Administration (US EIA) database. Weekly Cushing, OK Crude Oil Future Contract 1 was used as the proxy for oil futures. Weekly New York Harbor Reformulated RBOB Regular Gasoline Future Contract 1 was used as the proxy for gasoline futures prices. Weekly New York Harbor Conventional Gasoline Regular Spot Price FOB was used as the proxy for gasoline spot prices. Weekly New York Harbor No. 2 Heating Oil Future Contract 1 was used as the proxy for heating oil futures. Weekly New York Harbor No. 2 Heating Oil Spot Price FOB was used as the proxy for heating oil spot prices.

For WTI, weekly spot and futures prices over the 3rd January 1986 to 8th April 2016 period were used; this produced 1580 observations per variable. For gasoline prices, weekly spot and futures data over the 3rd March 2006 to 8th April 2016 period were used; this produced 528 observations per variable. For heating oil prices, weekly spot and futures data over the period 6th June 1986 to 8th April 2016 were used: this resulted in 1558 observations per variable.

4.0 METHODOLOGY
4.1 Computing the OHR

The empirical analysis of this study is divided into two phases. Phase one involved pretesting. A log transformation is applied to all variables that are subsequently tested for stationarity using the Augmented Dickey Fuller (ADF), the Phillips Perron (PP), the Perron (1997) and Zivot and Andrews (1992) tests. Where the data were found to be non-stationary, the spot and futures prices for each commodity were tested for cointegration.

The Engle Granger two-step method, the Johansen method, the Phillips–Ouliaris method and an augmentation of the Gregory Hansen method are used to test for Cointegration. Phase two involved the use of econometric models to estimate the OHR. The following methods were used to compute the OHR: OLS, ARDL-ECM, DCC-GARCH-ECM, and the Kalman Filter. An interested reader is referred to Appendix 1 for a detailed discussion of the OLS, ARDL, GARCH and the Kalman Filter methodologies.

In keeping with the objective of this study, the opitmal hedge is estimated by the methods of OLS, ARDL-ECM, Copula-EGARCH, and the Kalman Filter to facilitate a comparison of the effectiveness of the hedging the methods employed.

4.2 Hedging effectiveness

This study proposes a new method to assess hedging effectiveness (HE) based on changing the OHR into a portfolio weight.In a two asset portfolio of spot and futures, the portfolio weight for spot would be the percentage of spot in the portfolio. Likewise, the portfolio weight for futures is the percentage of futures in the portfolio. The two weights must sum to unity. If the portfolio weights are a function of the OHR, then they should directly change as the OHR changes.
Hammoudeh et al. (2009) proposed that the optimal portfolio weight should be given by:

$$w_{sf,t} = \frac{h_{f,t} - h_{sf,t}}{h_{s,t} - 2h_{sf,t} + h_{f,t}} \qquad (13)$$

$$w_{sf,t} = \begin{cases} 0 & if w_{sf,t}, 0 \\ w_{sf,t}\,, & if\ 0 < w_{sf,t} < 0 \\ 1 & if w_{sf,t} > 0 \end{cases} \qquad (14)$$

where $w_{sf,t}$ is the weight of spot, $1 - w_{sf,t}$ is the weight of futures, $h_{f,t}$ is the variance of futures, $h_{s,t}$ is the variance of spot, and $h_{sf,t}$ is the covariance of spot and futures.

However, such optimal portfolio weight do not consider the OHR that was estimated via different methods. This study proposes the weight for spot will be $w_s = \frac{1}{(1+OHR)}$ and the weight for futures will be $w_f = \frac{OHR}{(1+OHR)}$. Such weights were choosen because the OHR indicates how much units of futures should be used to offset one unit of spot. Assume that x units of futures were estimated via the OHR. Then such a portfolio should have 1 unit of spot and x units of futures. Therefore, the amount of spot in that portfolio would be $\frac{1}{1+x}$ while the amount of futures in that portfolio would be $\frac{x}{1+x}$. After the portfolio weight is estimated, the variance of the portfolio can be computed using the following formula:

$$\sigma_p^2 = \sqrt{(w_s^2\sigma_s^2 + (1 - w_s)^2\sigma_f^2 + 2w_s(1 - w_s)c\rho_{s,f}\sigma_s\sigma_f)} \qquad (15)$$

Equation (15) is the Variance of the Portfolio formula with ρ replaced by the Copula (cρ) which captures a better dependence than correlation (de Melo et al. 2004; Wang et al. 2009; Wang and Cai 2011). This study proposes that the model that computes the OHR and produces the smallest variance will have the highest hedging effectiveness. This logic is consistent with minimizaton of the portofolio hedgeing strategy (McAleer 2010).

5.0 EMPIRICAL RESULTS
5.1: Pretesting

The results from the ADF, PP, Peron stationarity tests are presented in Table 1.The results suggest that all the prices are non-stationary with a unit root.

Table 1: Stationarity tests results

	ADF level	ADF 1st difference	PP level	PP 1st difference	P	ZA
WTI spot	0.3654	0.0000	0.4635	0.0000	-4.62	0.0085
WTI futures	0.5639	0.0000	0.5101	0.0000	-4.06	0.0053
Gasoline spot	0.1098	0.0000	0.1638	0.0000	-4.10	0.0018
Gasoline futures	0.1029	0.0000	0.1263	0.0000	-4.06	0.0028
Heating oil spot	0.4080	0.0000	0.4353	0.0000	-3.61	0.0062
Heating oil futures	0.4596	0.0000	0.4741	0.0000	-3.59	0.0044

The data was tested for ARCH effects to verify if a volatility model would be relevant for modelling. In the test for ARCH effects, the null hypothesis states that no ARCH effects exist. The alternative hypothesis states that ARCH effects exist. For each return, the probabilities of the F statistic and the Chi-Square statistic were less than the 1% significance level. Such results lead to the rejection of the null hypothesis and indicated that each return has ARCH effects; accordingly, GARCH type modeling is appropriate for modeling the optimal hedge ratio (OHR) (Table 2).

Table 2: ARCH effects

Variable	Test Statistic	Probability
WTI spot returns	Prob. F(1,1577)	0.0000
	Prob. Chi-Square(1)	0.0000
WTI futures returns	Prob. F(1,1577)	0.0000
	Prob. Chi-Square(1)	0.0000
Gasoline spot returns	Prob. F(1,525)	0.0000
	Prob. Chi-Square(1)	0.0000
Gasoline futures returns	Prob. F(1,525)	0.0000
	Prob. Chi-Square(1)	0.0000
Heating oil spot returns	Prob. F(1,1555)	0.0000
	Prob. Chi-Square(1)	0.0000
Heating oil futures returns	Prob. F(1,1555)	0.0000
	Prob. Chi-Square(1)	0.0000

Since the data is non-stationary, it is tested for cointegration. The results from the cointegration test presented in Table 3 suggest that spot and futures prices are cointegrated for WTI, gasoline, and heating oil, respectively. Such cointegration result implies that cointegration type modelling is relevant for the spot and futures data.

Table 3: Cointegration results

	EG	PO	Johansen1 coint vect.	GHThe critical values used for the GH test are the critical values from the LS test.
WTI spot and futures	0.0000	0.0000	0.2086	-34.05
gasoline spot and futures	0.0000	0.0000	0.1040	-6.15
Heating oil spot and futures	0.0000	0.0000	0.1347	-10.90

5.2 Estimation of OHR, OPW and Variance of Portfolio using different methods

The OLS, ARDL-ECM, Copula-EGARCH, and the Kalman Filter all suggested that the Naïve Hedging Strategy is the most appropriate for hedging crude oil (Table 4). Since the hedging effectiveness method used in this study is based on the OHR, all methods employed had the same hedging effectiveness for WTI. Thus, for crude oil, $1 of futures can be used to hedge $1 of spot. Such result will produce an OPW close to 0.5 since 50% of the portfolio will be held by spot, and another 50% will be held by futures. The OHR results for WTI imply that the futures market is functioning efficiently, since the OHR will be close to unity if the market is efficient.

In the rows illustrating the OHR, the number is brackets is the $\bar{R}^2$.Appendix 2, illustrates how the OHR was computed using the Copula EGARCH.

With regards to gasoline, the Copula-EGARCH method produced the lowest variance of the portfolio and thus had the highest hedging effectiveness. The OLS method had the second highest hedging effectiveness and the ARDL-ECM had the lowest hedging effectiveness. Since the ARDL-ECM also recommended an OHR greater than unity, it had recommended an over hedged ratio. The Copula-EGARCH method recommended an OHR of 0.7261 implying that $0.73 in futures is required to offset every dollar held in the spot position for gasoline.

For heating oil, the Copula-EGARCH also had the highest hedging effectiveness. The ARDL-ECM had the lowest hedging effectiveness. Since the ARDL-ECM also estimated an OHR greater than unity, it recommended an over hedged ratio. The OLS and the Kalman Filter had the same hedging effectiveness since they produced an OHR of 0.99, which is relatively close to the naïve hedge. Consequently, $0.77 should be held in futures to offset every dollar held in spot for heating oil.

For gasoline, the OLS model suggested an OHR of 0.91. The ARDL-ECM suggested an OHR of 1.76. In fact, the ARDL-ECM results for gasoline and heating oil implied an over hedge. The Kalman Filter results for all variables wereclose to the Naïve Hedging Strategy. The Copula-EGARCH suggested an OHR of 0.72 for gasoline, and 0.74 for heating oil. The Copula-EGARCH had the highest hedging effectiveness as it had the smallest portfolio variance. Subsequently, $0.72 should be held in futures to optimally hedge every dollar held in spot for gasoline. Also, $0.74 should be held in futures to optimally hedge every dollar held in spot for heating oil.

Table 4: OHR, OPW and Variance of Portfolio using different methods.

		WTI	Gasoline	Heating oil
OHR	OLS	0.99 (0.9308)	0.91 (0.8106)	0.99 (0.8410)
	ARDL-ECM	1.00 (0.9815)	1.76 (0.7250)	1.07 (0.9987)
	Copula-EGARCH	0.9993	0.7261	0.7742
	Kalman Filter	0.99	0.98	0.99
		WTI	Gasoline	Heating oil
OPW	OLS	0.4974 (futures)	0.4764 (futures)	0.4974 (futures)
	ARDL-ECM	0.5 (futures)	0.6377 (futures)	0.5169 (futures)
	Copula-EGARCH	0.4998 (futures)	0.4207 (futures)	0.4364 (futures)
	Kalman Filter	0.4974 (futures)	0.4949 (futures)	0.4974 (futures)
		WTI	Gasoline	Heating oil
Variance of Portfolio	OLS	16.6568	0.3315	0.3769
	ARDL-ECM	16.6568	0.3482	0.3797
	Copula-EGARCH	**16.6568**	**0.3255**	**0.3274**
	Kalman Filter	16.6568	0.3334	0.3769

6.0 CONCLUSIONS

Crude oil and its refinery fraction prices are volatile. Though upward price movement is favored by the producer of the commodities, negative price movement is highly undesirable. The literature has identified hedging with futures contracts as one possible method for a risk adverse commodity producer to protect itself from commodity price risk.

WTI spot was shown to have a high dependence onWTI futures. In addition, all methods recommended an OHR close to the Naïve Hedge Strategy. Thus, all four methods yielded similar hedging effectiveness.

For gasoline and heating oil, the results identified the Copula EGARCH to be the superior method since it produced the lowest variance of the portfolio.

Although Kalman Filter was recommended to compute the OHR in Wang et al. (2010), our results show that it is not the most effective method in the refinery fraction industry. The Copula-EGARCH was proven to be the best method for computing the OHR for the refinery fraction industry. The Copula-EGARCH recommends an OHR close to the naïve hedge for WTI; 0.7261 for gasoline; and 0.7742 for heating oil. The Copula-EGARCH recommended an OPW of 0.4998 for WTI futures; 0.4207 for gasoline futures and 0.4364 for heating oil futures.

Such findings in this study have implications for policy. Companies producing and exporting gasoline and heating oil may use the OHR estimated from the Copula-EGARCH to take the short term position. However, such hedges would be favorable when the prices of gasoline and heating oil are declining. When the price of the refinery fractions prices rebound, such hedges should not be undertaken since hedging would reduce their profitability.

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APPENDIX1

Detailed Review of Methodology used to estimate the OHR

In this study multiple models: the OLS, ECM, GARCH, and the Kalman Filter models were used to estimate the OHR for crude oil, gasoline, and heating oil.

Model 1: Classical Linear Regression Model

OLS is a well-known model. For details see Brooks (2008). The OLS method is a linear regression of changes in spot prices on changes in futures prices. It will take the following form:

$\Delta S_t = \alpha_0 + \beta \Delta F_t + \varepsilon_t$ (A.01)

whereΔS_t is the first difference of spot prices, α_0 is an intercept, ΔF_t is the first difference of futures prices, β is the OHR and ε_t is an error term.

Model 2: ARDL-ECM

ARDL-ECM is an Auto Regressive Distributive Lag (ARDL) model with an error correction term to account for cointegration. An ARDL is similar to an OLS model; however, it contains lags of variables, whereas an OLS do not contain lags. The ARDL-ECM deployed takes the following form:

$\Delta S_t = \alpha_0 + \lambda \Delta F + \beta_1 \Delta S_{t-1} + \beta_2 \Delta F_{t-1} + \theta_1 z_{t-1} + \varepsilon_t$ (A.02)

where λ is the OHR, ΔS_{t-1} is the lag of the first difference of spot prices, ΔF_{t-1} is the lag of the first difference of futures prices, z_{t-1} is the error correction term, β_1, β_2, and θ_1 are respective coefficients.

Model 3: Copula-GARCH

The GARCH model is a method used to model the time varying conditional variance of a time series. A GARCH (1,1) model is given by:

$\sigma^2 = \alpha_0 + \alpha_1 u_{t-1}^2 + \beta \sigma_{t-1}^2$ (A.03)

whereσ^2 is the conditional variance; α_0 is the meannewsabout volatility from the previous period. Kenourgios et al. (2001) stated that the OHR will also be α_0. The ARCH term is u_{t-1}^2which is also a lag of the error term from the mean equation; σ_{t-1}^2 is a lag of the conditional variance (the GARCH term); α_1 and β are coefficients of the lag of the squared error and lagged conditional variance respectively, which were obtained by estimation. If $\alpha_1 + \beta \geq 1$ then shocks take long to die out.If $\alpha_1 + \beta = 1$, then there is a unit root in the GARCH model. In such a case an Integrated GARCH (IGARCH) model can be used to model the volatility.

The GARCH (q,p) model fails to capture asymmetric effects. Also a restriction need to be imposed upon the GARCH to prevent the conditional variance from becoming negative if its estimated $\alpha_1$and β coefficients are large and negative. To correct such limitations, Nelson (1991) proposed an Exponential GARCH (EGARCH) model given by:

$ln(\sigma^2) = \omega + \beta \ln(\sigma_{t-1}^2) + \gamma \frac{u_{t-1}}{\sqrt{\sigma_{t-1}^2}} + \alpha \left[\frac{|u_{t-1}|}{\sqrt{\sigma_{t-1}^2}} - \sqrt{\frac{2}{\pi}} \right]$ (A.04)

However EVIEWS modifies equation (A6.04) to:

$\log(\sigma^2) = \omega + \sum_{i=1}^{q} \alpha_i \left| \frac{u_{t-i}}{\sigma_{t-i}} \right| + \sum_{k=1}^{r} \lambda_k \frac{u_{t-k}}{\sigma_{t-k}} + \sum_{j=1}^{p} \beta_j \log(\sigma_{t-j}^2)$ (A.05)

where$ln(\sigma^2)$ is the log transformation that has been applied to the variance; ω is a constant; α is the coefficient associated from the size/magnitude of shock effect, λ is the coefficient associated with the sign or leverage effect, and β is the coefficient for the persistence of shocks (the GARCH term). If the leverage effects are positive (negative) and statistically significant, then negative (positive) shocks will increase volatility more than positive (negative) shocks of the same magnitude.

The Constant Conditional Correlation (CCC)-GARCH is a multivariate GARCH. It is typically a preferred to the BEKK GARCH model since it is parsimonious, while the BEKK has a lot of parameters to estimate. The CCC-GARCH model first estimates two univariate GARCH model for two variables. Then it takes the standardized residuals from each GARCH volatility estimation to estimate correlation. In the CCC-GARCH, the researcher specifies

$h_{s,t}^2 = \alpha_{0s} + \alpha_s \varepsilon_{s,t-1}^2 + \beta_s h_{s,t-1}^2$ (A.06)

$h_{f,t}^2 = \alpha_{0f} + \alpha_f \varepsilon_{f,t-1}^2 + \beta_f h_{f,t-1}^2$ (A.07)

$H_t = \begin{bmatrix} h_{s,t}^2 & h_{sf,t} \\ h_{sf,t} & h_{f,t}^2 \end{bmatrix} = \begin{bmatrix} h_{s,t} & 0 \\ 0 & h_{f,t} \end{bmatrix} \begin{bmatrix} 1 & \rho \\ \rho & 1 \end{bmatrix} \begin{bmatrix} h_{s,t} & 0 \\ 0 & h_{f,t} \end{bmatrix} = D_t R D_t$ (A.08)

whereS_t is spot prices, F_t is futures prices, $h_{s,t}^2$ is the variance of S_t, $h_{f,t}^2$ is the variance of F_t, R is the correlation matrix, and $D_t = \begin{bmatrix} h_{s,t} & 0 \\ 0 & h_{f,t} \end{bmatrix}$. Also

$OHR = \frac{h_{sf,t}}{h_{f,t}^2}$ (A.09)

The limitation of the CCC-GARCH is its assumption that the correlation will be constant. Correlation between two series may change with time as posited by Engle and Sheppard (2001). Engle and Sheppard (2001) and Engle (2002) subsequently proposed the Dynamic Conditional Correlation (DCC)-GARCH which is almost identical to the CCC-GARCH. It is given by

$H_t = D_t R D_t$ (A.10)

However, its correlation matrix R is time varying. In other words the DCC-GARCH estimates two correlation matrices for the difference time periods to join the two univariate GARCH models.

The DCC-GARCH and CCC-GARCH models are limited in that they are based on correlation. Under extreme market conditions, correlation based measures and measures based on the Gaussian distribution are unreliable (Malevergne and Sornette2006).Additionally,correlation based measures can yield spurious relationships (Aas 2004; Malevergne and Sornette 2006).Thus, what is really needed in finance is models that can perform well under both normal market conditions and extreme market conditions.

One such model is a copula. A copula essentially “joins together” different distributions. Copulas were introduced in financial applications by Embrechts et al. (1999). Since then, they have been widely used. Some of their applications can be seen by the works of Embrechts et al. (2002), de Melo et al. (2004), Dias and Embrechts (2004).

According to Sklar (1959), every p-dimensional distribution F that has a marginal distribution F_i, has a copula C, such that:

$F(x_1, \ldots., x_p) = C(F(x_1), \ldots, F(x_p))$ (A.11)

According to Vogiatzoglou (2010) this may be rewritten as:

$C(u_1, \ldots, u_p) = F(F_1^{-1}(u_1), \ldots, F_p^{-1}(u_p))$ (A.12)

where$u_i = F_i(x_i)$, and i ranges from 1 to p, p is the dimension

If the function F is p-times differentiable, then the joint function may be derived by:

$f(x) = \frac{\partial p}{\partial x_1 \partial x_2 \ldots \partial x_p} F(x)$ (A.13)

$= \prod_{i=1}^{p} f_i(x_i) \frac{\partial p}{\partial x_1 \partial x_2 \ldots \partial x_p} C(F(x_1), \ldots, F(x_p))$ (A.14)

where f(x) is the joint density function. Thus

$f(x) = \prod_{i=1}^{p} f_i\,(x_i)\; C(F(x_1), \ldots, F(x_p))$ (A.15)

The copula density function that is obtained is:

$c(u_1, \ldots, u_p) = \frac{f(F_1^{-1}(u_1), \ldots, F_p^{-1}(u_p))}{\prod_{i=1}^{p} f_i(F_p^{-1}(u_i))}$ (A.16)

where$x = (x_1, \ldots, x_p)$ and the copula density function is c.

The parameters of the copula may be estimated by the optimization of the log-likelihood function

$L(\xi; x) = \sum_{j=1}^{T} (\sum_{i=1}^{P} log\,(f_i(x_{i,t}; \phi_i)) + \log\,(c(F_1(x_{1,t}), \ldots, F_p(x_{p,t}); \theta))$ (A.17)

where $\xi = (\phi, 0)$. It is the vector that holds the copula estimates θ and the marginal distribution estimates ϕ. The marginal estimates likelihood function is given by

$mL(\phi; x) = \sum_{j=1}^{T} \sum_{i=1}^{P} log\,(f_i(x_{i,t}; \phi_i)$ (A.18)

The copula likelihood function is given by:

$cL(\theta; u; \phi) = \log\,(c(F_1(x_{1,t}), \ldots, F_p(x_{p,t}); \theta))$ (A.19)

In this study, a Copula-EGARCH is used to join the marginal distributions from the GARCH type models into a joint distribution. The Copula-GARCH is comprised of multiple series that were modelled by a univariate GARCH model, and they were later joined together with a Copula. Copula – GARCH models have also been used in finance (Embrechts et al. 2001, Panchenko 2006, Serban et al. 2007, Huang et al. 2009 and Wang et al. 2009; 2011). Additionally, Lai et al. (2009) have used a Copula-GARCH model to compute an OHR.

The EGARCH model is specified and the residuals are obtained. The residuals are then converted into a uniform distribution. The marginal distributions are then joined into a joint distribution using a Copula. There are multiple types of Copulas, each vary depending of the type of dependence they represent. The Student-t distribution Copula could be used rather than a Gaussian distribution Copula because a lack of normality was found for each series, and thus the normality assumption was not made. Also, the Guassian Copula has no dependence in the tails, while the Student-t Copula has dependence in the tails (Embrechts et al. 2001; Jondeau and Rockinger2006).

The Student-t Copula allows for the joining of marginal distributions in extreme events, but not for asymmetries. The tail dependence in the Student-t copula is symmetric (Wang and Cai, 2011). The Clayton copula, which is an asymmetric copula, is a better choice to model marginal distributions with negative asymmetries (Vogiatzoglou 2010). Another type of asymmetric Copula is the Gumbel copula that is better suited for modeling asymmetries in the positive tail (Vogiatzoglou, 2010).The Clayton Copula and the Gumbel Copula belong to the Archimedean copula family.Other Archimedean copulas include: Frank, Joe and Ali-Mikhail-Haq(Shams and Haghighi, 2013).copulas allow for the modeling of dependence in high dimensions with only one parameter (Shams and Haghighi, 2013). In Chapter 5 only the Student-t Copula is used because it can perform better under extreme market conditions than the Gaussian Copula. Additionally, oil prices tend to fluctuate in both positive and negative directions. Since it is relatively unknown when oil

price would increase or decrease, the Student-t Copula is used for its asymmetric dependence. Table A6.01 highlights the equations for various Copulas.
For the Gaussian Copula, ρ is the estimated parameter andΦ^{-1} is the inverse of the Gaussian function. For the Student-t Copula, ρ and v are estimated parameters and$t_v^{-1}$is the inverse of the Student-t function. For the Clayton Copula, $0 < \delta < \infty$ is the parameter that controls the dependence in the negative tail. For the Gumbel Copula, $0 < \delta \leq 1$ is the parameter that controls the dependence in the positive tail.

Table A.01: Characteristic Equations for various Types of Copulas

Type of Copula	Equation
Gaussian	$C_\rho(u,v) = \int_{-\infty}^{\Phi^{-1}(u)} \int_{-\infty}^{\Phi^{-1}(v)} \frac{1}{2\pi(1-\rho^2)^{1/2}} \exp\left\{-\frac{x^2 - 2\rho xy + y^2}{2(1-\rho^2)}\right\} dxdy$
Student-t	$C_{\rho,v}(u,v) = \int_{-\infty}^{t_v^{-1}(u)} \int_{-\infty}^{t_u^{-1}(v)} \frac{1}{2\pi(1-\rho^2)^{1/2}} \left\{1 + \frac{x^2 - 2\rho xy + y^2}{v(1-\rho^2)}\right\}^{-(v+2)/2} dsdt$
Clayton	$C_\delta(u,v) - (u^{-\delta} + v^{-\delta}\ 1)^{-1/\delta}$
Gumbel	$C_\delta(u,v) - \exp\left(-\left[(-\mathrm{logu})^\delta + (-logv)^\delta\right]^{\frac{1}{\delta}}\right)$

Source:Compiled from Vogiatzoglou(2010)

Model 4: Kalman Filter

The Kalman Filter is a mathematical algorithm that can be used to separate the noise (random variations) from the signal in data (Welch, and Bishop, 1995). It produces estimates of variables that tend to be reliable and accurate. A KalmanFilter is an optimal estimator in that it makes estimates based on uncertain data. It finds the best estimate from noisy data by filtering away the noise. It is also recursive;consequently, measurements are processed as they arrive to find the optimal value for the next state. Notably, it remembers information about the previous states.
The two equations of Kalman Filter are as follows:

$$\begin{cases} x_k = Ax_{k-1} + Bu_k + w_k \\ \quad z_k = Hx_k + v_k \end{cases} \qquad \text{(A.20)}$$

The signal x_k is a function of a linear combination of the signal in the previous period or state x_{k-1}, a control signalu_k, and a noise w_{k-1}. The x_k equation is the state prediction equation. The z_k equation is the sensor prediction equation. A, B, and H are coefficients that are estimated.
If the noise is Gaussian distributed, the Kalman Filter minimizes the mean squared error of its estimates. If the noise is not normally distributed, the Kalman Filter is still the best linear estimator. In real life signals may not be purely Gaussian distributed; however, normality is frequently assumed for estimation purposes.
The second part of equation (A6.20) tells us that the measured value z_k is a function of the signal, and the measurement noise.Theprocessnoisew_{k-1} is statistically independent of the measurement noise v_k.
As the Kalman Filter is specified, it makes a time prediction about the states ahead, namely:
$x_k = Ax_{k-1} + Bu_k$, and the error covariance ahead $P_k^- = AP_{k-1}A^T + Q$.
When new information comes in it makes a measurement update as the predicted states ahead would not be 100% accurately forecasted. Thus the predicted state ahead would differ from the actual state ahead. The next step involves computing the Kalman Gain $K_k = \frac{P_k^- H^T}{(HP_k^- H^T + R)}$
It updates the measured value estimates $\hat{x}_k = \hat{x}_k^- + K_k(z_k + H\hat{x}_k^-)$
and the error covariance $P_k = (1 - K_k H)P_k^-$. The new output for P_k will enter the equation $P_k^- = AP_{k-1}A^T + Q$ as the P_{k-1} to create the forecast for the next state of P_{k+1}.
The Kalman Filter can be represented by:

$$\hat{x}_k = \hat{x}_k^- + K(z_k - H\hat{x}_k^-) \qquad \text{(A.21)}$$

where the estimate $\hat{x}_k$will be a linear combination of the forecasted estimate $\hat{x}_k^-$, and a multiple of the difference between the actual measurement z_k and the predicted measurement $H\hat{x}_k^-$:the multiplier is called the Kalman gain (K).
If the predicted measurement is close to the actual measure, then$(z_k - H\hat{x}_k^-)$ will be close to 0 and multiplication by the Kilman gain (K) will be small or insignificant. If however, the actual measurement if far different from the predicted

measurement, the Kalman gain will tell us by how much to correct it. Thus $(z_k - H\hat{x}_k^-)$ can be considered as a correction term. The difference $(z_k - H\hat{x}_k^-)$ is also referred to as the measurementinnovation (Welch, and Bishop, 1995).
Thus, the whole idea of the Kalman Filter is to incorporate information about our previous states, the actual measurement value/sensor and the predicted measurement to make an accurate prediction for the next state.
To estimate the time varying hedge ratio with the Kalman Filter, the following system is specified:

$$\begin{cases} S_t = \propto + \beta f_t + \varepsilon_t \\ \beta_t = \beta_{t-1} + v_t \end{cases} \qquad \text{(A.22)}$$

The first equation in (A6.22) is the signal or observation equation while and the second equation is the state or transition equation. The state equation shows the time varying of the hedge ratio that follows an autoregressive process of order 1. The residual terms $\varepsilon_t$and v_t are assumed to be Independent and Identically Distributed (I.I.D).
Equation (A6.22) may be specified as

$$\begin{cases} Y_t = \propto + \beta X_t + \varepsilon_t \\ \beta_t = A\beta_{t-1} + \eta_t \end{cases} \qquad \text{(A.23)}$$

whereY_t is an N ×1 vector that is used to model spot prices, X is an N × k matrix that may include futures prices and any other explanatory variable, β_t is a k x 1 vector of the coefficients that were estimated. Likewise, $\varepsilon_t \sim N(0, \sigma_\varepsilon)$, $\eta_t \sim N(0, \eta_\varepsilon)$, and $E(\varepsilon_t, \eta_t) = 0$.
The next step involves the estimation of the parameters A, Q and P and makinginferenceabout the time varying coefficient β. Thus the following equations are specified:

$$\hat{\beta}_{t|t-1} = A\hat{\beta}_{t-1} \qquad \text{(A.24)}$$

$$P_{t|t-1} = AP_{t|t-1}A' + Q \qquad \text{(A.25)}$$

$$\varepsilon_t = (y_t - \hat{\beta}'_t x_{t|t-1}) \qquad \text{(A.26)}$$

$$f_t = x'_t P_{t|t-1} x_t + \sigma_\varepsilon \qquad \text{(A.27)}$$

$$\hat{\beta}_t = \hat{\beta}_{t|t-1} + P_t x_{t|t-1} \left[\frac{\varepsilon_t}{f_t}\right] \qquad \text{(A.28)}$$

$$P_t = P_{t|t-1} x_t x'_t P_{t|t-1} \left[\frac{1}{f_t}\right] \qquad \text{(A.29)}$$

where$\hat{\beta}_t$ was estimated via maximum likelihood estimation, P_t is the variance of $\hat{\beta}_t$, ε_t denotes the one step ahead prediction error which has a variance of f_t. The subscript $t|t-1$ is the conditional estimation of parameters at time t, given information at the previous period t-1.

APPENDIX 2

Steps used to compute the OHR for WTI, gasoline and heating oil using Copula-EGARCH

The following illustrates the steps for the computation for the OHR for WTI, gasoline and heating oil using Copula-EGARCH

$$WTI_{OHR} = \begin{bmatrix} h_{s,t}^2 & h_{sf,t} \\ h_{sf,t} & h_{f,t}^2 \end{bmatrix} = \begin{bmatrix} 16.6533 & 0 \\ 0 & 16.662 \end{bmatrix} \begin{bmatrix} 1 & 0.9998 \\ 0.9998 & 1 \end{bmatrix} \begin{bmatrix} 16.6533 & 0 \\ 0 & 16.662 \end{bmatrix}$$

$$h_{s,t}^2 = 16.6533 \; x \; 16.6533 \quad = 277.332$$

$$h_{f,t}^2 = 16.662 \; x \; 16.662 = 277.622$$

$$h_{sf,t} = 16.6533 \; x 0.9998 \; x \; 16.662 = 277.421$$

$$h_{sf,t} / h_{f,t}^2 = 277.421 / 277.622 = 0.9993$$

$$gasoline_{OHR} = 0.2836 * 0.9884 * 0.3862 / 0.3862 * 0.3862 = 0.7261$$

$$heatingoil_{OHR} = 0.4261 * 0.9988 * 0.3303 / 0.4261 * 0.4261 = 0.7742$$